A Causa of Computation Error Caused by an Linear Interpolation Method

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Abstracts : Transient simulators such as EMTP, ATP and EMTDC include an algorithm of a piece-wise linear modeling to express a nonlinear characteristic. The piece-wise linear algorithm with the linear interpolation method causes a large error in an oscillating frequency and its attenuation when it is applied to a resonance circuit including a nonlinear element. Because the operational region of the linear interpolation is determined by the result of the previous calculation, a wrong conductance of a nonlinear element or a wrong current source expressing a linear inductance and capacitance around a nonlinear element is used for the calculation at the transitions.

This paper proposes a method of improving the computation error using the combined iterative method. The iterative conductance gives an accurate oscillating frequency and its attenuation. The conductance is determined without any interpolation in a time step loop, and is useful to achieve a high accuracy in simulating an oscillation circuit including a nonlinear element.

A simulation of an amplitude modulation circuit with a diode is presented to show the effectiveness of the proposed algorithm, and the calculated results are compared with an ATP result, an EMTDC result and a measured result. The comparison proves the validity of the proposed algorithm to analyze an oscillation circuit, which is difficult to get high accuracy with faster simulation time.

1. INTRODUCTION

Transient simulators such as EMTP, ATP [1] and EMTDC [2] are generally used in the field of power engineering. Dommel's method [1], which is a basic formula of those transient simulators, expresses every circuit element as a parallel connection of a linear conductance and a current source. The simulators include an algorithm of a piece-wise linear modeling to express a nonlinear characteristic such as an arrester. An arc model in Ref [3] and a diode model in Ref [4] are notable examples using the piece-wise linear expression. Though the piece-wise linear modeling is a current approach for a nonlinear modeling in a transient analysis to reduce its simulation time, its accuracy should be improved.

The piece-wise linear algorithm with the linear interpolation method [5] causes a large error in an oscillating frequency and its attenuation when it is applied to a resonance circuit including a nonlinear element. Because the operational region of the linear interpolation is determined by the result of the previous calculation, a wrong conductance of a nonlinear element or a wrong current source expressing a linear inductance and capacitance around a nonlinear element is used for the calculation at the transitions. Though the scheme has been applied to accelerate the calculation and to increase the accuracy, the transitions cause the errors.

This paper proposes a method of improving the computation error using the combined iterative method [6]. The iterative conductance gives an accurate oscillating frequency and its attenuation. The conductance is determined without any interpolation in a time step loop, and is useful to achieve a high accuracy in simulating an oscillation circuit including a nonlinear element.

A simulation of an amplitude modulation circuit with a diode is presented to show the effectiveness of the proposed algorithm, and the calculated results are compared with an ATP result, an EMTDC result and a measured result. The comparison proves the validity of the proposed algorithm to analyze an oscillation circuit, which is difficult to get high accuracy with faster simulation time.

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2. CIRCUIT EQUATION AND PIECE-WISE LINEAR APPROXIMATION

When all elements in a circuit are linear, the elements are described by trapezoidal rule of integration [1], and the circuit equation includes only linear conductances and linear current surces depend on past circuit informations, and can be expressed in the following matrix equation:

$$\mathbf{J}(t-\Delta t)=\mathbf{G}\cdot\mathbf{v}(t) \qquad (1)$$

where $\mathbf{v}$: node voltage vector, $\mathbf{G}$: constant conductance matrix for all time steps, $\mathbf{J}$: time varying current source vector. The triangular factorization of $\mathbf{G}$ is performed only once before advancing to a time step loop, and $\mathbf{v}(t)$ is calculated by forward and backward substitutions. At the end of each time step, $\mathbf{J}(t)$ is renewed to calculate $\mathbf{v}(t+\Delta t)$ which is the node voltage vector at a next time step.

When the circuit includes some nonlinear elements using a nonlinear conductance, a nodal-conductance matrix G depends on many factors such as an instantaneous node voltage solution $\mathbf{v}(t)$ and branch current $\mathbf{i}(t)$. The nonlinear circuit equation can be expressed as the following matrix equation:

$$\mathbf{G}(t, \mathbf{v}(t), \mathbf{i}(t)\,\mathbf{v}(t))=\mathbf{J}(t-\Delta t)+\mathbf{J}_{non}(t, \mathbf{v}(t), \mathbf{i}(t)) \qquad (2)$$

where $\mathbf{G}$: nonlinear conductance matrix, $\mathbf{J}$: time varying current source vector for linear elements and actual sources, $\mathbf{J}_{non}$: nonlinear current source vector for nonlinear elements. As before, the retriangulation of $\mathbf{G}$ in eq. (2) is required whenever the factors such as a branch voltage and current of a nonlinear element change. If it is possible to express *v*-*i* characteristic of the nonlinear elements by using piece-wise linear approximation, the retriangulation is not necessary to solve eq. (2).

3. LINEAR INTERPOLATION

A linear interpolation method can be described with an example of a simple diode as a switch model and a piece-wise linear one.

The waveform in Fig.1 shows the current through the diode as a switch with a standard fixed time-step switching algorithm. The current reverses at a time in between Δt and $2\Delta t$, but because of the discrete nature of time-step, the impedance of the devise can only be made at $t=\Delta t$ and $2\Delta t$. The first recorded instant of zero current is thus at $3\Delta t$.

Fig.2 shows the same device with the diode as a switch interpolated to correct instant. As before, the program calculates the solution at $t=\Delta t$ and $t=2\Delta t$. However, on noticing that at latter time, the current has already crossed zero, it estimates the turn-off time to be $t=1.2\Delta t$ based on a linear interpolation of the current within the switching interval. All the voltage and current in the trapezoidal solution method are then also interpolated to this intermediate time in a linear fashion. However, it should be noted that the current injections of linear elements such as an inductance and a capacitance can not be modified at $t=1.2\Delta t$, the current injection at $t=1.2\Delta t$ may be equal to the one at t $=1.0\Delta t$ in EMTDC using a linear interpolation. The conductance matrix is then re-formulated and the solution continues with the original time step, yielding the new solution one time step later at $t=2.2\Delta t$. One additional interpolation step between $t=1.2\Delta t$ and $t=2.2\Delta t$ yields the solution at $t=2\Delta t$. The later apparently cosmetic step is taken to put the solution back on the original time grid.

Fig.3 shows the diode as a piece wise linear model interpolated to correct instant.

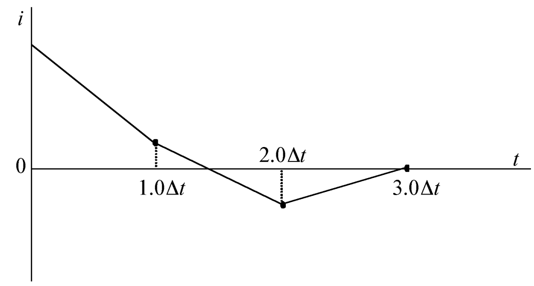

Fig.1 Current Waveform without Interpolation

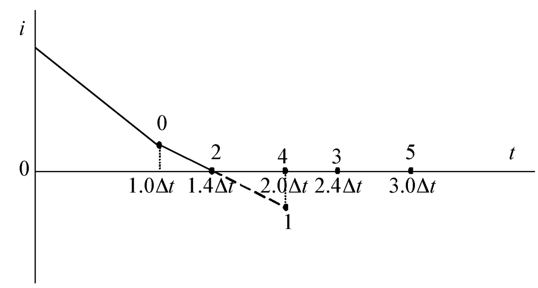

Fig.2 Current Waveform with Interpolation

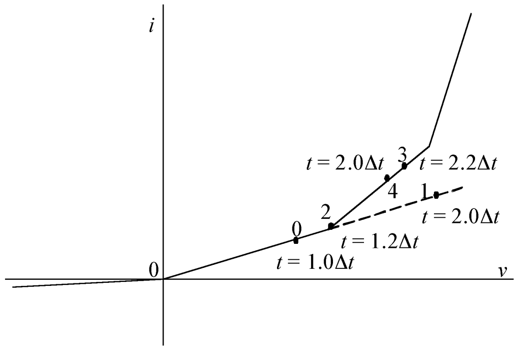

Fig.3 Diode *v-i* Characteristic with Interpolation

4. COMBINED ITERATIVE METHOD

4.1 Modified Predictor Corrector Iterative Method

The combined iterative method (CIM) employs two kinds of iterations efficiently. One of the iterative methods in CIM is the Modified Predictor Corrector Iterative (MPCI) method [6, 7]. The solution $\mathbf{v}^{(0)}(t)$ of the following equation gives the first estimation of the iteration (prediction).

$$\begin{aligned}&\mathbf{G}\,(\,t,\,\mathbf{v}(\,t-\Delta t\,),\,\mathbf{i}(\,t-\Delta t\,))\,\mathbf{v}^{(0)}(t)\\&\quad=\mathbf{J}\,(\,t-\Delta t\,)+\mathbf{J}_{non}(\,t,\,\mathbf{v}(\,t-\Delta t\,),\,\mathbf{i}(\,t-\Delta t\,))\end{aligned}\tag{3}$$

The improved solutions are repeatedly obtained by the following iteration scheme (correction).

$$\begin{aligned}&\mathbf{G}\,(\,t,\,\mathbf{v}^{(k-1)}(t),\,\mathbf{i}^{(k-1)}(t))\,\mathbf{v}^{(k)}(t)\\&\quad=\mathbf{J}\,(\,t-\Delta t\,)+\mathbf{J}_{non}(\,t,\,\mathbf{v}^{(k-1)}(t),\,\mathbf{i}^{(k-1)}(t))\end{aligned}\tag{4}$$

where $k = 1, 2, \ldots$: the number of iterations. It should be noted that MPCI method doesn't require a lot of reconstitutions of a conductance matrix **G** at each time step and each iterative step, because a nonlinear element is expressed as a piecewise linear conductance and a nonlinear current injection.

4.2 Newton Raphson Method

The multidimensional root finding method by NRI Method is discussed in [8]. NRI gives us a very efficient means of converging to a root, if a sufficiently good initial value can be guessed. If it fails to converge, it indicates that the roots of the solution do not exist nearby.
A typical problem gives N functional relations to be zeroed, which involves variables x_i, $i = 1, 2, \ldots$,N.

$$Fi\,(\,x_1,\,x_2,\,\ldots,\,x_N\,)=0\tag{5}$$

Each of the functions Fi in eq. (5) can be expanded in Taylor series,

$$F_i(\,\mathbf{x}+\delta\mathbf{x}\,)=F_i(\mathbf{x})+\sum_{j=1}^{N}\frac{\partial F_i}{\partial x_j}\delta x_j+O(\delta x^2)\tag{6}$$

where $\mathbf{x}$: entire vector of values x_i, $\mathbf{F}$: entire vector of functions Fi. The matrix of partial derivatives appearing in eq. (6) is Jacobian matrix **Jc**.

$$Jc_{ij}=\frac{\partial F_i}{\partial x_j}\tag{7}$$

In matrix notation, eq. (6) is written by:

$$\mathbf{F}\,(\,\mathbf{x}+\delta\mathbf{x}\,)=\mathbf{F}(\mathbf{x})+\mathbf{Jc}\cdot\delta\mathbf{x}+O(\delta\mathbf{x}^2)\tag{8}$$

By neglecting the term of order $\delta\mathbf{x}^2$ and higher, and by setting $\mathbf{F}(\mathbf{x}+\delta\mathbf{x})=\mathbf{0}$, a set of linear equations for correction $\delta\mathbf{x}$, which moves each function closer to zero simultaneously, is derived as following equation [9].

$$\mathbf{Jc}\cdot\delta\mathbf{x}=-\mathbf{F}\tag{9}$$

Effective construction of Jacobian matrix is discussed in Ref. [9].

4.3 Combined Iterative Method

The CIM employs the MPCI method and NRI one efficiently. Subsidiary techniques such as an optimum ordering of nodes in the nonlinear conductance matrix **G** in eq. (2), an unified expression method of eq. (4) and (9), an efficient handling method of eq. (2) using Crout's algorithm and an effective adoption of the iterations are proposed Ref.[6], those method are very important to realize the CIM in EMTP-type simulators.

5. AMPLITUDE MODULATION CIRCUIT

5.1 Relations between Input and Output

An amplitude modulation circuit including a diode as an example is illustrated in Fig.4. The diode model for CIM is approximated as the following equation.

$$i=av^2+bv\tag{10}$$

where i : current [A], v : voltage [V], a = 1.28×10^{-6}, b = 0.003×10^{-6}. When a carrier wave $c(t)=A\sin\omega_c t$ and a signal wave $s(t)=S\sin\omega_s t$ are input, the filtered output can be derived as follows.

$$i_{AM} = (Ab + 2aAS\sin\omega_c t)\sin\omega_c t \tag{11}$$

5.2 Actual Measurement

A germanium diode has been used in Fig.4, the diode voltage has been impressed between about ±0.3 [V] and the maximum current of about 0.6 [mA] has followed through the diode.

The *v-i* characteristic of the germanium diode for piece-wise linear model used in EMTDC and EMTP simulation is shown in Table.1. The experimental result of the output voltage in Fig 4 is shown in Fig.5. The frequency and the amplitude of the carrier wave are 1.44 MHz and 0.408 V respectively, and the frequency and

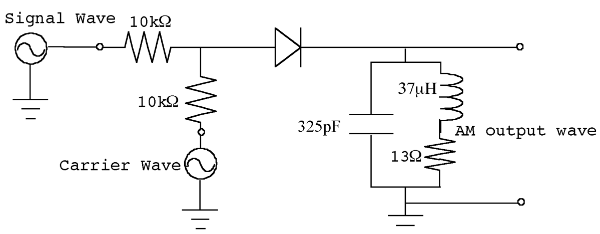

Fig.4 Amplitude Modulation Circuit Including a diode

Table 1 v-i caracteristics of a germanium diode

電流（μA）	電圧（V）
−51	−0.2
−50	−0.2
−29	−0.1
0	0
0.0012	0.01
0.03	0.05
0.097	0.09
0.2	0.13
0.27	0.15
0.35	0.17
0.43	0.19
0.53	0.21
0.6	0.25
2.5	0.4
6	0.6

the amplitude of the signal wave are 980 Hz and 0.210 V respectively. The carrier and signal wave can be expressed as following equations.

Carrier wave : $c(t) = 0.408\sin(2\times\pi\times1.44\times10^{6}\times t)$ (12)

Signal wave : $s(t) = 0.210\sin(2\times\pi\times980\times t)$ (13)

5.3 Calculated Results

The EMTDC solution using the interpolation method in Fig.6, the EMTDC solution avoiding the use of the interpolation method in Fig.7, the ATP solution in Fig.8 and the CIM solution in Fig.9 are compared with the experimental result of output voltage shown in Fig.5. A 0.05 ms time step has been used for all simulations. The frequency and the amplitude of the carrier wave are 1.44 MHz and 0.408 V respectively, and the frequency and the amplitude of the signal wave are 980 Hz and 0.210 V respectively.

The EMTDC software does not have a piece-wise liner approximation model for a diode. Therefore, the germanium diode model alternated a switch diode model and a piece-wise liner arrester model has been used to describe the curve shown in Table 1 in the EMTDC simulations.

In the ATP simulation, Type-92 as a piecewise linear diode model has been used to describe the curve shown in Table 1.

In the CIM simulation, a nonlinear element is expressed as a parallel connection of a piecewise linear conductance and a nonlinear current source for MPCI, and a parallel connection of a nonlinear conductance and a nonlinear current source for NRI, because of the diminution in error between the quadratic function diode model in eq.(10) and a piece-wise linear approximation [6.9].

5.4 Comparisons

The EMTDC result with the interpolation method has a margin of error in the amplitude as compared with all other calculated results and experimental result. It should be noticed in the EMTDC simulation that such-like result puts in an appearance at a general oscillatory circuit including a nonlinear element expressed by a piecewise linear approximation. The reason of these is

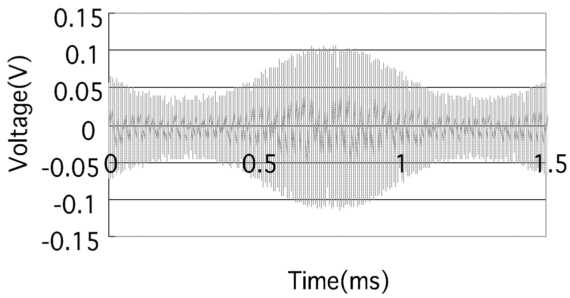

Fig. 5 Experimental Result

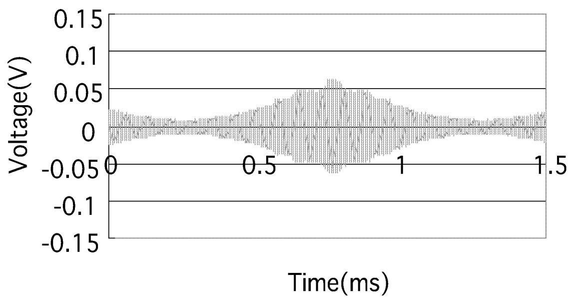

Fig. 6 EMTDC Result with Interpolation Method

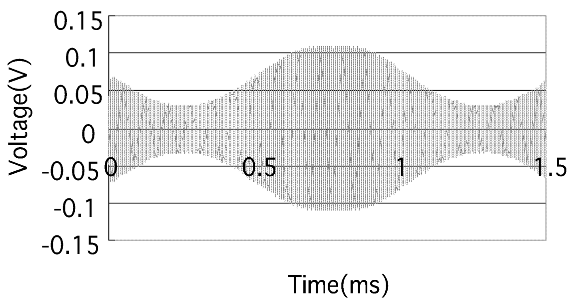

Fig. 7 EMTDC Result without Interpolation Method

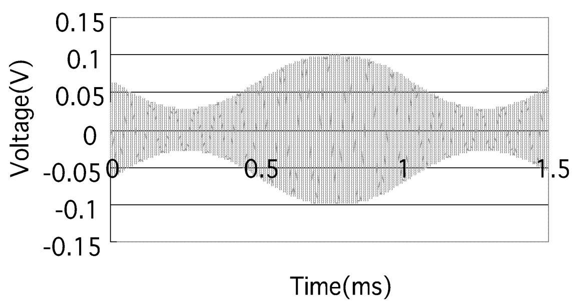

Fig. 8 ATP result

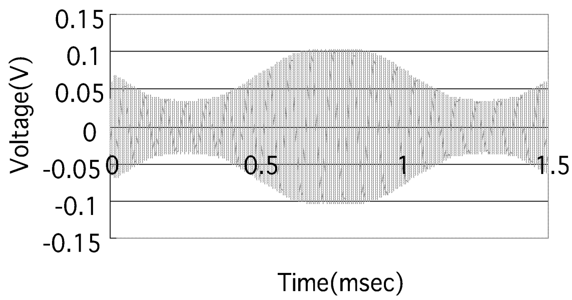

Fig. 9 CIM result

thought of as the error arising from erroneous current source expressing linear capacitance and inductance elements at t = $1.4\Delta t$ in Fig.2 and $t = 1.2\Delta t$ in Fig.3, because the current sources at $t = 1.4\Delta t$ in Fig.2 and $t = 1.2\Delta t$ in Fig.3 are alternated with the one at $t = 1.0\Delta t$ in Fig.2 and Fig.3. As another reason, because the operational region of the linear interpolation is determined by the result of the previous calculation, a wrong conductance may be used for the calculation at the transitions.

The EMTDC result without the interpolation method in Fig.7 and ATP result in Fig.8 are practically same with the experimental result. However, small error of the amplitude of output appears around $t = 0.3$ [ms] and $t = 1.3$ [ms].

The CIM result in Fig.9 agrees well with the experimental result. The conductance expressing a germanium diode is determined without any interpolation in a time step loop, and is useful to achieve a high accuracy in simulating an oscillation circuit including a nonlinear element.

6. CONCLUSIONS

It has been submitted that the piece-wise linear algorithm with the linear interpolation method causes a large error in an oscillating frequency and its attenuation when it is applied to a resonance circuit including a nonlinear element. This paper has been offered the reasons and the solutions for the problem, and CIM has been proposed to realize more accurate simulation of such circuit in EMTP-type simulators.

A simulation of an amplitude modulation circuit with a diode is presented to show the effectiveness of the proposed algorithm. The EMTDC solution using the interpolation method, the EMTDC solution avoiding the use of the interpolation method, the ATP solution and the CIM solution are compared with the experimental result. The EMTDC result without the interpolation method and EMTP result are practically same with the experimental result, but have small error. CIM result agrees well with the experimental result.

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