An Optimum Adoption of Iterative Methods for Nonlinear Simulations on EMTP-Type Simulators

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Abstract - This paper adopts a combined iterative method (CIM) as a basis of a transient calculation for nonlinear circuits. CIM consists of a modified predictor corrector iteration (MPCI) and Newton-Raphson iteration (NRI) algorithms, and a parallel connection of a piecewise linear conductance and a nonlinear current source for MPCI and a parallel connection of a nonlinear conductance and a nonlinear current source for NRI. It is important to define an adoption of iterative methods in CIM, because a solution obit depends on the multi-dimensional plane between an initial and real solutions.

This paper presents an optimum adoption of iterative methods MPCI and NRI which are basis of CIM for nonlinear simulations, an effective construction of MPCI and Jacobian matrices based on a multi-dimensional solution with Dommel's method and an effective expression of MPCI and NRI on EMTP-Type simulators. This paper also demonstrates the proposed algorithm on an example system, and compares the result obtained with a basic nodal conductance approach (NCA). The results prove the validity of those proposed methods for any kinds of EMTP-Type simulators with nonlinear elements.

Keywords: Nonlinear, Combined Iterative Method, Modified Predictor Corrector Iteration, Newton-Raphson Iteration, Optimum Adoption, Optimum handling, EMTP-type Simulator.

I. INTRODUCTION

A nodal-conductance approach (NCA) with Dommel's method [1] is a basis of many Electro-Magnetic Transients Programs such as EMTP, ATP [2] and PSCAD/EMTDC [3]. In this paper, the simulators based on Dommel's method are defined as EMTP-type simulators. A piecewise linear approximation model of a nonlinear element is a main current in the field of electromagnetic transients simulators, where a nonlinear element can be modeled by a parallel connection of a linear conductance and a linear current injection. However, the expression may make some restrictions to model a complicated nonlinear element such as fault arc models [4] and power electronics models [5] (thyristor, diode, GTO and IGBT), which are utilized in a simulation of a complicated system such as HVDC and FACTS. A restriction of arrangements of nonlinear elements [6] is the one of those in the simulators, and instability and inaccuracy caused by the piecewise modeling and the arrangement can also result when applied to the complicated system.

A linear interpolation [7,8] can represent simple nonlinear devices using piecewise linear approximations, and results in stable and accurately calculated results for any number of the nonlinear devices. However, it may be difficult to express all kinds of nonlinear devices in this way. Therefore it is important to complete a more general, accurate and stable representation of the nonlinear elements.

CIM [9] is adopted in this paper as a basis of a transient calculation of nonlinear circuits. CIM consists of MPCI and NRI algorithms, and a parallel connection of a piecewise linear conductance and a nonlinear current source for MPCI and a parallel connection of a nonlinear conductance and a nonlinear current source for NRI. Previous works in this area are based on Newton-Raphson method in SPICE [10] and non-iterative method [2] in ATP, but each has restrictions of the number and configuration of the nonlinear elements, and involves some problems such as instability, low efficiency, inaccuracy in convergency. It is important to define an adoption of iterative methods in CIM, because a solution obit depends on the multi-dimensional plane between an initial and real solutions.

This paper presents an optimum adoption of iterative methods which (MPCI and NRI) for nonlinear simulations, an effective construction of MPCI and Jacobian matrices based on a multi-dimensional solution with Dommel's method and an effective expression of MPCI and NRI on EMTP-Type simulators. This paper also demonstrates the proposed algorithm on an example system, and compares the result obtained with a basic NCA. The results prove the validity of those proposed methods for any kinds of EMTP-Type simulators with nonlinear elements.

II. ITERATIVE EQUATIONS

The basis of CIM is explained in this chapter briefly. It is verified that basic formulas of MPCI and NRI method for a nonlinear transient simulation can be expressed as a same equation using Dommel's method and CIM basis, and the formulations make an effective construction and handling of nonlinear matrixes possible as in section III.

A. Modified Predictor Corrector Iteration

One of the iterative methods in CIM is MPCI method. MPCI method doesn't require a lot of reconstitutions of a conductance matrix G at each time step and each iterative step, because a nonlinear element is expressed as a piecewise linear conductance and a nonlinear current injection.

The solution $\mathbf{v}^{(0)}(t)$ of the following equation gives the first estimation of the iteration (prediction).

$$\mathbf{G}(t)\mathbf{v}^{(0)}(t) = \mathbf{J}(t) + \mathbf{J}(t, \mathbf{v}(t-\Delta t)) \tag{1}$$

The improved solutions are repeatedly obtained by the following iteration scheme (correction):

$$\mathbf{G}(t)\mathbf{v}^{(k)}(t) = \mathbf{J}(t) + \mathbf{J}(t, \mathbf{v}^{(k-1)}(t)) \tag{2}$$

where k = 1, 2, ... : the number of iterations.

B. Newton-Raphson Iteration

NRI gives a very efficient means of converging to a root, if a sufficiently good initial value can be guessed. If it fails to converge, it indicates that the roots of the solution do not exist nearby.

A typical problem gives N functional relations to be zeroed, which involves variables x_i, i = 1, 2, ... ,N.

$$F_i(x_1, x_2, \ldots, x_N) = 0 \qquad i = 1, 2, \ldots, \mathrm{N}. \tag{3}$$

Each of the functions Fi in eq. (3) can be expanded in Taylor series:

$$F_i(\mathbf{x}+\delta\mathbf{x}) = F_i(\mathbf{x}) + \sum_{j=1}^{N} \frac{\partial F_i}{\partial x_j}\delta x_j + O(\delta\mathbf{x}^2) \tag{4}$$

where $\mathbf{x}$: entire vector values x_i, $\mathbf{F}$: entire vector of functions F_i. The matrix of partial derivatives appearing in eq. (4) is Jacobian matrix **Jc**:

$$Jc_{ij} = \frac{\partial F_i}{\partial x_j} \tag{5}$$

In matrix notation, eq. (4) is written by:

$$\mathbf{F}(\mathbf{x}+\delta\mathbf{x}) = \mathbf{F}(\mathbf{x}) + \mathbf{Jc}\cdot\delta\mathbf{x} + O(\delta\mathbf{x}^2) \tag{6}$$

By neglecting the term of order $\delta\mathbf{x}^2$ and higher and by setting F($\mathbf{x}+\delta\mathbf{x}$) = 0, a set of linear equations for correction $\delta\mathbf{x}$, which moves each function closer to zero is derived simultaneously.

$$\mathbf{Jc}\cdot\delta\mathbf{x} = -\mathbf{F} \tag{7}$$

Eq. (7) in electrical circuits can be solved efficiently by LU decomposition. A correction is then added to the solution vector $\mathbf{x}$.

$$\mathbf{x}_{new} = \mathbf{x}_{old} + \delta\mathbf{x} \tag{8}$$

In an EMTP-type simulator, Jacobian matrix including some nonlinear elements can be constructed in an example with a nonlinear element between node i and j efficiently [9]. Circuit vector **F** in eq. (7) is expressed as follows :

$$\begin{aligned} G_{ii} &= G_{Lii} + G_N \quad G_{ij} = G_{Lij} + G_N \\ G_{ji} &= G_{Lji} + G_N \quad G_{jj} = G_{Ljj} + G_N \\ J_i &= J_{Li} + J_N \quad J_j = J_{Lj} + J_N \end{aligned} \tag{9}$$

$$\mathbf{F} = \begin{bmatrix} G_{11} & \cdots & G_{1i} G_{1j} \\ \vdots & \ddots & \vdots \ \vdots \\ G_{i1} & \cdots & G_{ii} G_{ij} \\ G_{J1} & \cdots & G_{ji} G_{jj} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ V_j \end{bmatrix} - \begin{bmatrix} J_1 \\ \vdots \\ J_i \\ J_j \end{bmatrix} \tag{10}$$

where G_L: conductance of linear elements, G_N: conductance of a nonlinear element, J_L: current source of linear elements, J_N: current source of a nonlinear element. From above relations, following Jacobian Matrix can be derived [9].

$$G_{Lii}+\frac{dI}{dV_i}=G_{Lii}+\frac{df}{dV},\ G_{Lij}+\frac{dI}{dV_j}=G_{Lij}-\frac{df}{dV},$$
$$G_{Lji}-\frac{dI}{dV_i}=G_{Lji}-\frac{df}{dV},\ G_{Ljj}-\frac{dI}{dV_j}=G_{Ljj}+\frac{df}{dV} \tag{11}$$

$$\frac{\partial \mathbf{F}}{\partial \mathbf{V}}=\begin{bmatrix} G_{11} & \cdots & G_{1i} & G_{1j} \\ \vdots & \ddots & \vdots & \vdots \\ G_{i1} & \cdots & \left(G_{Lii}+\frac{df}{dV}\right) & \left(G_{Lij}-\frac{df}{dV}\right) \\ G_{i1} & \cdots & \left(G_{Lji}-\frac{df}{dV}\right) & \left(G_{Ljj}+\frac{df}{dV}\right) \end{bmatrix} \tag{12}$$

In eq. (12), Jacobian matrix can be efficiently constructed by differential function ***df/dv***, which can be calculated analytically or numerically. If the function f can be expressed analytically such as an arrester model, the differential function ***df/dv*** can often be calculated by analytical differentiation of f. If the function f cannot be expressed analytically as a fault arc model, the method to differentiate the function f can be calculated numerically.

C. Unified Expression of Iterative Equations

The feature of eq.(8) in NRI procedures make it difficult to save the calculation time of a nonlinear circuit, because an optimum handling explained in chapter III can not be applied. Therefore, it is important that MPCI and NRI equations eqs. (1), (2) and (7) which are basis of CIM can be expressed as a same equation using Dommel's method and CIM basis.

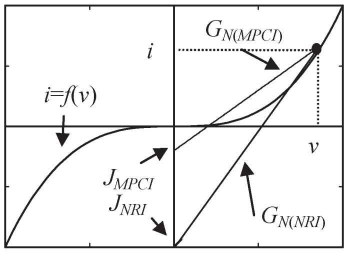
Eq. (10) is substituted for eq. (7), and the following equation can be derived.

$$\mathbf{Jc}\cdot\delta\mathbf{v}+\mathbf{Gv}=\mathbf{J} \tag{13}$$

As the derivations of eq. (12), an example circuit, which includes a nonlinear element between node i and j, is illustrated to compare the conductance matrix **G** with Jacobian matrix **Jc** in eq. (13). When a nonlinear element is composed of a piecewise linear conductance and a nonlinear current injection, the conductance matrix can be expressed as follows.

$$G(t)=\begin{bmatrix} G_{11} & \cdots & G_{1i} & G_{1j} \\ \vdots & \ddots & \vdots & \vdots \\ G_{i1} & \cdots & (G_{Lii}+G_N) & (G_{Lij}-G_N) \\ G_{j1} & \cdots & (G_{Lji}-G_N) & (G_{Ljj}+G_N) \end{bmatrix} \tag{14}$$

Because the piecewise linear conductance G_N of a nonlinear element is explained as in Fig. 1, it means that G_N is equivalent to ***df/dv*** in eq. (12). Therefore, in CIM procedures, both MPCI and NRI equations can be constructed in the same eq. (1) and (2), but it should be notice that the expression of MPCI conductance, of which a nonlinear element is composed, is piecewise linear, not nonlinear as NRI conductance.



$$i=G_{N(NRI)}v+J_{NRI}\quad \textit{for NRI Method}$$
$$i=G_{N(MPCI)}v+J_{MPCI}\quad \textit{for MPCI Method}$$

Fig.1 Difference of MPCI and NRI conductance

III. HANDLING OF MATRIXES

A. Ordering of Nodes

When all elements in the circuit are linear, those elements is described by the trapezoidal rule of integration [1] in the following matrix equation:

$$\mathbf{G}_L\,\mathbf{v}(t)=\mathbf{J}(t) \tag{15}$$

where $\mathbf{G}_L$: linear (constant) nodal-conductance matrix, $\mathbf{v}(t)$: node voltage vector, and $\mathbf{J}(t)$: linear current injection vector. To save calculation time, the triangular factorization of $\mathbf{G}_L$ is performed only once before advancing to the time step loop, and $\mathbf{v}(t)$ is calculated by the backward substitution. At the end of each time step, $\mathbf{J}(t)$ is renewed to calculate $\mathbf{v}(t+\Delta t)$.

When the circuit involves nonlinear elements, $\mathbf{G}_L$ depends on some factors as $\mathbf{v}(t)$. Thus, the retriangulation of $\mathbf{G}_L$ is required whenever the factors are changed at each time step and each iterative step [11].

The retriangulation of $\mathbf{G}_L$ in MPCI is not required at each time step and iterative step, because the nonlinear elements are expressed as a piecewise linear conductance and a nonlinear current injection. For the retriangulation of Jacobian matrix in eq. (7) is required at each iterative step in the NRI method, the effective computation of Jacobian Matrix applied the unified expression in chapter II becomes important.

The optimum ordering for the optimum handling method is illustrated in Fig.2.

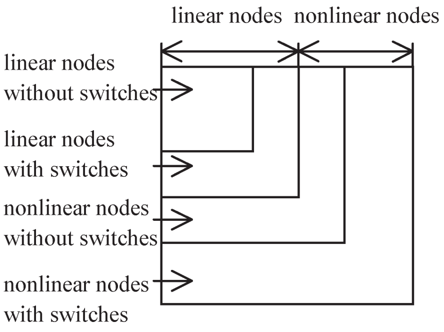


Fig. 2 Optimum ordering of nodes in **G**

B. Optimum Handling

(1) The advantage of LU decomposition

Matrix **A** can be written as a product of two matrices **L** and **U** (lower and upper triangular matrices respectively). When we solve the linear set by the following decomposition [12],

$$\mathbf{A}\cdot\mathbf{x}=(\mathbf{L}\cdot\mathbf{U})\cdot\mathbf{x}=\mathbf{L}\cdot(\mathbf{U}\cdot\mathbf{x})=\mathbf{L}\cdot\mathbf{y}=\mathbf{b} \tag{16}$$

$\mathbf{L}\cdot\mathbf{y}=\mathbf{b}$ in eq. (16) is solved to get the vector **y**. Then $\mathbf{U}\cdot\mathbf{x}=\mathbf{y}$ is solved to get the real solution vector **x**.

An advantage of breaking up one linear set into two successive ones is that the solution of a triangular set of equation is quite trivial. Thus, eq. (16) can be solved by forward and backward substitution.

$$y_1=\frac{b_1}{L_{11}},\quad y_i=\frac{1}{L_{ii}}\left[b_i-\sum_{j=1}^{i-1}L_{ij}y_j\right]\quad i=2,3,\cdots,N \tag{17}$$

$$x_N=\frac{y_N}{U_{NN}},\ x_i=\frac{1}{U_{ii}}\left[y_i-\sum_{j=i+1}^{N}U_{ij}x_j\right]\quad i=N-1,\cdots,1 \tag{18}$$

The most important advantage for EMTP-type simulators is that once we have the LU decomposition of **A**, we can solve with as many right-hand sides **b** without reconstructions of **A**.

(2) Crout's algorithm for LU decomposition

Very efficient procedure is Crout's algorithm [12], which quite trivially solves $\mathbf{A}=\mathbf{L}\cdot\mathbf{U}$ by just arranging the equations in the following order.

a) Set $L_{ii}=1$, $i=1,\ldots,N$.

b) For each $j=1,2,3,\ldots,N$: First, for $i=1,2,\ldots,j$, use the following equation to solve for U_{ij},

$$U_{ij}=A_{ij}-\sum_{k=1}^{i-1}L_{ik}U_{kj}, \tag{19}$$

Second, for $i=j+1, j+2,\ldots,$N use eq. (20) to solve for L_{ij},

$$L_{ij}=\frac{1}{U_{jj}}\left(A_{ij}-\sum_{k=1}^{j-1}L_{ik}U_{kj}\right) \tag{20}$$

Note that the both procedures have to be done before going on to the next j.

It is obvious from the above that L and U on the right-hand side of eqs. (19) and (20) are already determined by the time when those are needed. Every A_{ij} is used only once and never again, i.e. the corresponding L_{ij} or U_{ij} can be stored in the location where the A_{ij} used to occupy. In brief, Crout's method fills in the combined matrix of **L** and **U** by columns from left to right, and within each column from top to bottom.

(3) Relations between LU decomposition and Norton-Thevenin equivalent for nonlinear matrix equations

For an efficient and fast calculation of the nonlinear conductance and Jacobian matrices, an equivalent circuit in Fig.3 is proposed. Once the equivalent circuit is determined, it is very efficient and fast to calculate the nonlinear conductance and Jacobian matrices, because linear nodes in Fig.3 needs not to be changed during the whole time steps. To define this equivalent circuit automatically for every kind of electromagnetic circuits, a partial LU decomposition and a partial forward and backward substitution based on Crout's algorithm are proposed in this paper.

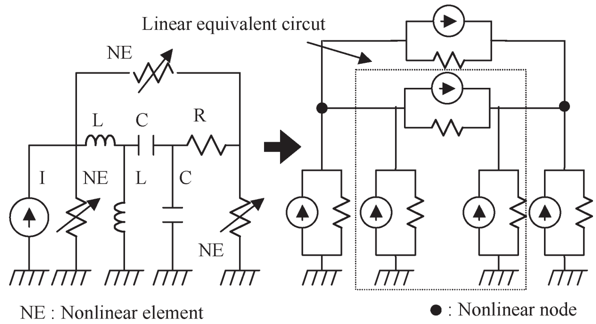


Fig. 3 Equivalent circuit for a nonlinear circuit

Node 1, 2,..., M are linear nodes and node M+1,..., N are nonlinear nodes on a general nonlinear conductance and Jacobian matrices.

a) Set $L_{ii}=1$, $i=1,\ldots,$N.

b) For each $j=1,2,\ldots,$M where M is the maximum number of linear nodes : First, for i = 1,2,...,j, use eq. (19) for U_{ij}. Second, for i $=j+1, j+2,\ldots,$N use eq. (20) for L_{ij}.

c) For $j=$ M+1 : For $i=1,2,\ldots,$M, use eq. (19).

From the above steps a), b) and c), the nonlinear matrix **A** is renewed to the following matrix form.

$$\begin{bmatrix} U_{11} & \cdots & U_{11M} & U_{1M+1} & \cdots & A_{1N} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ L_{M1} & \cdots & U_{MM} & U_{MM+1} & \cdots & A_{MN} \\ L_{M+11} & \cdots & L_{M+1M} & A_{M+1M+1} & \cdots & A_{M+1N} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ L_{N1} & \cdots & L_{NM} & A_{NM+1} & \cdots & A_{NN} \end{bmatrix} \tag{21}$$

It is important that renewed elements U_{11}, L_{21}, L_{31},...., U_{M-1M+1}, U_{MM+1} are constructed from the linear elements of **A**. Therefore, the renewed elements are constant for the whole simulation time steps and is performed only once before advancing to the time step loop.

d) The linear part of **y** ($y_1, y_2,..., y_M$) is calculated from the renewed elements U_{11}, L_{21}, L_{31},...., U_{M-1M+1}, U_{MM+1} by use of eq. (17), and also this part is constant and is performed only once before advancing to the time step loop.

e) The nonlinear part of **y** ($y_{M+1}, y_{M+2},..., y_N$) and the nonlinear part of **x** ($x_N, x_{N-1},..., x_{M+1}$) are calculated iteratively by CIM procedure.

f) After convergence, the nonlinear part of **x** ($x_M, x_{M-1},..., x_1$) is calculated, and we can proceed to next time step.

During iterative terms, the following equations ($\mathbf{L_n} \cdot \mathbf{y_n} = \mathbf{b_n}$ and $\mathbf{U_n} \cdot \mathbf{x_n} = \mathbf{y_n}$) are executed for the nonlinear conductance matrix as a partial forward and backward substitution.

$$\begin{bmatrix} L_{M+1M+1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ L_{NM+1} & \cdots & L_{NN} \end{bmatrix} \begin{bmatrix} y_{M+1} \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} b_{M+1} - \sum_{i=1}^{M} L_{M+1i} y_i \\ \vdots \\ b_N - \sum_{i=1}^{M} L_{Ni} y_i \end{bmatrix}$$
$$\begin{bmatrix} U_{M+1M+1} & \cdots & U_{M+1N} \\ \vdots & \ddots & \vdots \\ 0 & \cdots & U_{NN} \end{bmatrix} \begin{bmatrix} x_{M+1} \\ \vdots \\ x_N \end{bmatrix} = \begin{bmatrix} y_{M+1} \\ \vdots \\ y_N \end{bmatrix} \tag{22}$$

If $\mathbf{L_n} \cdot \mathbf{U_n}$ is symmetrical, eq. (22) shows that every kind of nonlinear circuits, which include M linear nodes and (N - M) nonlinear nodes, can be expressed as the equivalent circuit having only (N - M) nonlinear nodes as in Fig.4 which does not include any linear nodes. Actually

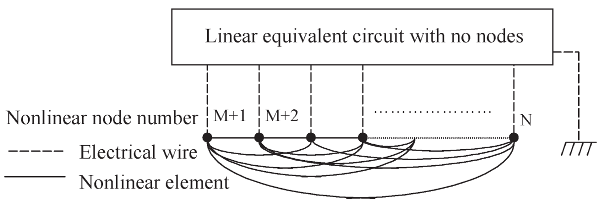


Fig. 4 Relation between LU decomposition and a nonlinear equivalent circuit

in all the kinds of electrical circuits on an EMTP-type simulator, $\mathbf{L_n} \cdot \mathbf{U_n}$ become symmetrical (see appendix A1 for details and verification). From eq. (22), an equivalent conductance matrix and an equivalent current injection vector can be $\mathbf{L_n} \cdot \mathbf{U_n}$ and $\mathbf{b_n}$ respectively.

(4) Comparison of the normal and partial method

A comparison of calculation times required for LU decomposition, backward and forward substitution and convergency is given in an example case in chapter V.

Basic NCA : 1.0
CIM without proposed method : 1.21
CIN with proposed method : 1.08

A difference of the calculation times heavily depends on the ratio of the number of nonlinear and linear nodes, but the proposed partial method is very efficient for treating not only the huge number of nonlinear nodes but also small number of nonlinear nodes. This method seems to become more powerful with the optimum sparse matrix method adopted in an EMTP-type simulator.

IV. EFFECTIVE ADOPTION OF ITERATIONS

NRI gives a very efficient means of converging to a root, if a sufficiently good initial value can be guessed. Therefore, it is very important to exploit how the improved solution is calculated from a first estimated solution which is not a sufficiently good initial value. So far, an effective adoption of iterations has not been discussed sufficiently, only methods of an iteration for nonlinear circuit have been researched.

CIM consists of two kinds of iterative methods, the most important feature of MPCI is very stable if a sufficiently good initial value can not be guessed, but the speed of convergency heavily depends on the direction of nonlinear conductances. On the other hands, the most important feature of NRI is that the speed of convergency is very fast, but the stableness heavily depends on the multi-dimensional plane between an initial and real solutions. Therefore, a standard of a good initial value need to be defined.

When the sign of first and second derivatives of nonlinear functions is not changed on the multi-dimensional plane between an initial and real solutions, the initial value is regarded as sufficiently good (region A in Fig.5), but if the sign is changed during convergency, MPCI method is employed to approach to a real solution (region B in Fig.5). Namely, if the first and second derivatives change their own sign during the way from solution x_i to x_{i+1} and from solution x_{i+1} to x_{i+2} continuously, MPCI method replaces NRI one from the next iterative term in CIM procedures. It means that a solution doesn't exist close to the solution x_i. If the first and second derivatives doesn't change continuously, NRI method has been used until convergency. After convergency by use of NRI method, MPCI method is employed once to make sure that the solution is real.

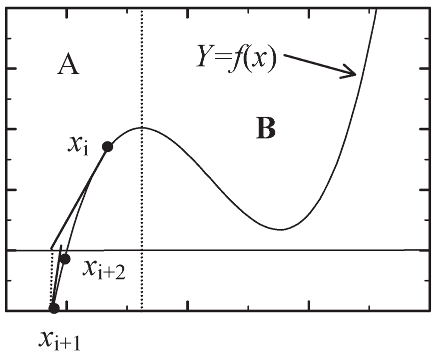


Fig. 5 Procces of convergency

V. EXAMPLE CASE

An oscillator circuit using a tunnel diode which is approximated in $i = 2.83v^3 - 2.02v^2 + 0.37v$ is adopted as an example case, which shows some advantages and the correctness of CIM in EMTP-type simulator. As other example cases for CIM (a diode rectifier circuit, an inverter circuit using transistors, a fault arc circuit and so on) have been illustrated in previous papers [4,9]. The oscillator circuit is shown in Fig.6. Simulated results with CIM adopting proposed schemes (Δt = 150, 5ns), and basic NCA results without iterations (Δt = 150, 5ns) are shown in Fig.7. As it can be seen in Fig.7, the CIM results show a close agreement with a calculated result with small dt, and the results obtained with CIM adopting proposed schemes are almost the same using different time steps.

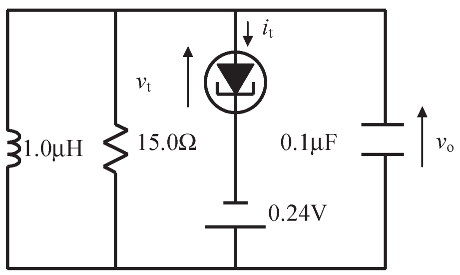
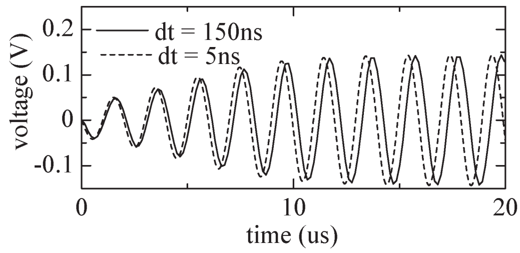
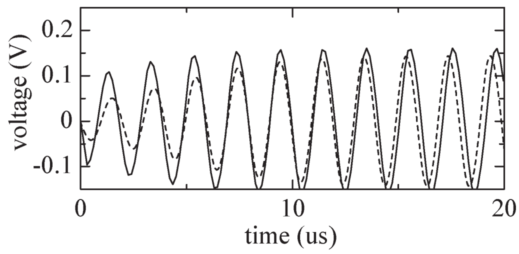


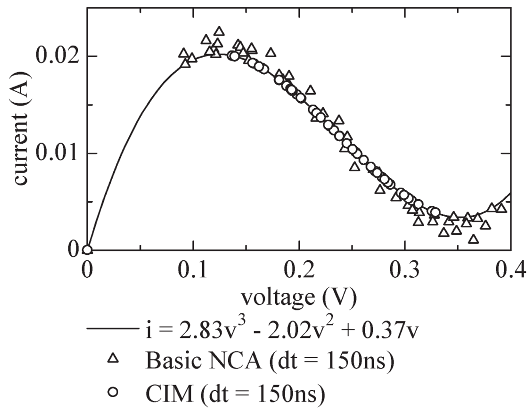
Fig. 6 Oscillator circuit using a tunnel diode



(a) Output voltage (CIM)



(b) Output voltage (basic NCA)



(c) v_t - i_t characteristic

Fig. 7 Calculated results

VI. CONCLUSIONS

Some developments of CIM solution method has been proposed to realize stableness and saving calculation time on an arbitrary number and configuration of nonlinear elements in a network. One of the developments is an optimum handling method of the nonlinear conductance matrix of CIM. The unified expression of nonlinear matrices has made it possible to save calculation time using the optimum handling based on the theory of

Crout's algorithm for a linear circuit. It has also proven that an effective adoption of iterations contributes to the stableness of CIM simulations.

The proposed methods have been applied to an oscillator circuit using a tunnel diode. Calculated results by the proposed method agree well with a theoretical result. The proposed methods have been confirmed to be accurate, fast and stable even for a large time step, and can survive from an abrupt change due to a nonlinear element.

The proposed methods can be easily implemented into an EMTP-type simulator because the methods are extensions of the basic NCA method in the EMTP-type simulator.

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VIII. APPENDIX

A1. *Proof of the symmetrical characteristic of the reduced nonlinear conductance matrix*

The pre-reduced nonlinear conductance matrix A, is completely symmetrical.

$$\begin{bmatrix} L_{11} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ L_{M1} & \cdots & L_{MM} & 0 & \cdots & 0 \\ L_{M+11} & \cdots & L_{M+1M} & L_{M+1M+1} & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ L_{N1} & \cdots & L_{NM} & L_{NM+1} & \cdots & L_{NN} \end{bmatrix} \begin{bmatrix} U_{11} & \cdots & U_{1M} & U_{1M+1} & \cdots & U_{1N} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & U_{MM} & U_{MM+1} & \cdots & U_{MN} \\ 0 & \cdots & 0 & U_{M+1M+1} & \cdots & U_{M+1N} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & U_{NN} \end{bmatrix} = \mathbf{A} \tag{A.1}$$

If we reduce the first line and column from eq. (A.1), the following equation is derived.

$$\begin{bmatrix} L_{22} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ L_{M2} & \cdots & L_{MM} & 0 & \cdots & 0 \\ L_{M+12} & \cdots & L_{M+1M} & L_{M+1M+1} & \cdots & 0 \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ L_{N2} & \cdots & L_{NM} & L_{NM+1} & \cdots & L_{NN} \end{bmatrix} \begin{bmatrix} U_{22} & \cdots & U_{2M} & U_{2M+1} & \cdots & U_{2N} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & U_{MM} & U_{MM+1} & \cdots & U_{MN} \\ 0 & \cdots & 0 & U_{M+1M+1} & \cdots & U_{M+1N} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & U_{NN} \end{bmatrix} = \mathbf{C} \tag{A.2}$$

From the symmetrical elements C_{ij} and C_{ji} in the matrix **C**, C_{ij} subtracted C_{ji} can be calculated as follows to prove that incidental matrix **C** is symmetrical.

$$\begin{aligned} C_{ij} - C_{ji} &= \sum_{k=2}^{i} L_{ik} U_{kj} - \sum_{k=2}^{i} L_{jk} U_{ki} \qquad (i < j) \\ &= A_{ij} - L_{i1} U_{1j} - \left(A_{ji} - L_{j1} U_{1i} \right) \\ &= L_{j1} U_{1i} - L_{i1} U_{1j} \quad \left(From\ A_{ij} - A_{ji} = 0 \right) \\ &= \frac{L_{i1}}{L_{11}} \left(L_{j1} U_{11} - U_{1j} L_{11} \right) \left(\begin{aligned} From\ A_{1j} &= L_{11} U_{1j} \\ &= A_{j1} = L_{j1} U_{11} \end{aligned} \right) \\ &= \frac{L_{i1}}{L_{11}} \left(A_{j1} - A_{1j} \right) = 0 \end{aligned} \tag{A.3}$$

Therefore, the reduced matrix **C** is symmetrical. Namely, reduced all matrices which are reduced some lines and columns from a symmetrical matrix are symmetrical.